Final-Math.R

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**PART A AND B**

library(alabama)

## Loading required package: numDeriv

# A and B ------------------------------------------------------------  
  
f <- function(x) {  
 return(60\*sum(x[1:5]) + 60\*sum(x[6:10]) + 60\*(x[1] + x[2] + x[3] + x[6] + x[7] + x[8] + x[11]) +  
 90\*(x[4] + x[5] + x[9] + x[10] + x[11] + 2\*x[12]))  
}  
  
g <- function(x) {  
 h <- numeric(14)  
 h[1] <- 6\*x[1]-32  
 h[2] <- 6\*x[6]-8  
 h[3] <- 6\*x[2]-68  
 h[4] <- 6\*x[7]-17  
 h[5] <- 6\*(x[1] + x[3])-56  
 h[6] <- 6\*(x[6] + x[8])-14  
 h[7] <- 6\*(x[2] + x[4])-76  
 h[8] <- 6\*(x[7] + x[9])-19  
 h[9] <- 6\*(x[3] + x[5] + x[11])-64  
 h[10] <- 6\*(x[8] + x[12])-16  
 h[11] <- 6\*(x[4] + x[11] + x[12])-28  
 h[12] <- 6\*x[9]-7  
 h[13] <- 6\*(x[5] + x[12])-8  
 h[14] <- 6\*x[10]-2  
 return(h)  
}  
  
p0 <- rep(20, 12)  
k <- constrOptim.nl(p0, f, hin = g)

## Min(hin): 52   
## par: 20 20 20 20 20 20 20 20 20 20 20 20   
## fval: 33000   
## Min(hin): 2.404391   
## par: 5.806799 12.2471 6.759881 1.301862 1.297652 2.163236 3.570814 2.26631 1.739286 0.7340652 4.201719 2.060152   
## fval: 5699.712   
## Min(hin): 0.04901609   
## par: 5.345459 11.98569 6.276006 0.7035728 0.8621676 1.354927 2.852702 2.055408 1.178759 0.3415027 3.569302 0.6543991   
## fval: 4700.511   
## Min(hin): 0.001037506   
## par: 5.333642 12.00438 6.348599 0.6628577 0.8208758 1.333879 2.833824 2.137813 1.16691 0.3335063 3.498226 0.5299488   
## fval: 4666.803   
## Min(hin): 2.546845e-05   
## par: 5.333343 12.00848 6.35775 0.6582038 0.816685 1.333346 2.833345 2.148024 1.166671 0.3333376 3.492252 0.5186724   
## fval: 4665.148   
## Min(hin): 7.075044e-07   
## par: 5.333334 12.00865 6.358816 0.6580192 0.8161531 1.333334 2.833333 2.149216 1.166667 0.3333372 3.491698 0.517452   
## fval: 4665.024   
## Min(hin): 5.895396e-07   
## par: 5.333334 12.00865 6.358816 0.6580192 0.8161531 1.333334 2.833333 2.149216 1.166667 0.3333372 3.491698 0.5174519   
## fval: 4665.024   
## Min(hin): 4.715748e-07   
## par: 5.333334 12.00865 6.358816 0.6580192 0.8161531 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174519   
## fval: 4665.024   
## Min(hin): 3.5361e-07   
## par: 5.333334 12.00865 6.358816 0.6580191 0.8161531 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174519   
## fval: 4665.024   
## Min(hin): 2.356452e-07   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 2.120522e-07   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 1.474394e-07   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 6.486408e-08   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 4.834901e-08   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 3.183393e-08   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 1.531888e-08   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 1.201585e-08   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 8.712831e-09   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 5.409817e-09   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 2.106802e-09   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024   
## Min(hin): 1.446196e-09   
## par: 5.333334 12.00865 6.358816 0.6580191 0.816153 1.333334 2.833333 2.149216 1.166667 0.3333371 3.491698 0.5174518   
## fval: 4665.024

k$par <- ceiling(k$par)  
print(k$par)

## [1] 6 13 7 1 1 2 3 3 2 1 4 1

print(f(k$par))

## [1] 5610

**Part A:  
How many full-time English-speaking agents, full-time Spanish-speaking agents, and part-time agents should Alex hire for each 2-hour shift to minimize operating costs while attending to all calls?**

Full-time English-speaking agents (7–9, 9–11, 11–1, 1–3, 3–5, 5–7, 7–9): 6, 12, 7, 1, 1, 0, 0

Full-time Spanish-speaking agents: 2, 3, 3, 2, 1, 0, 0

Part-time English-speaking agents: 0, 0, 0, 0, 0, 4, 1

**Part B:  
What is the minimum cost for the solution in Part A?**

Minimum Cost: $3620.00 (approximately)

**PART C AND D**

# C & D  
g2 <- function(x){  
 h <- numeric(2)  
 h[1] <- x[4] - 1  
 h[2] <- x[5] - 1  
 return(h)  
}  
  
# C AND D -----------------------------------------------------------------  
  
  
k <- constrOptim.nl(p0, f, hin = g, heq = g2)

## Min(hin): 52 Max(abs(heq)): 19   
## Outer iteration: 1   
## Min(hin): 52 Max(abs(heq)): 19   
## par: 20 20 20 20 20 20 20 20 20 20 20 20   
## fval = 33000   
##   
## Outer iteration: 2   
## Min(hin): 2.404492 Max(abs(heq)): 0.2963046   
## par: 5.8066 12.2514 6.76514 1.2963 1.28932 2.16409 3.57081 2.26257 1.73914 0.734082 4.20412 2.06496   
## fval = 5700   
##   
## Outer iteration: 3   
## Min(hin): 0.04878262 Max(abs(heq)): 0.2963046   
## par: 5.34545 12.0098 6.32161 0.679469 0.82602 1.35491 2.85275 2.0205 1.17869 0.341464 3.55982 0.689331   
## fval = 4701   
##   
## Outer iteration: 4   
## Min(hin): 0.0009867739 Max(abs(heq)): 0.2963046   
## par: 5.33364 11.943 6.36593 0.724228 0.843541 1.33388 2.83383 2.15918 1.16691 0.333498 3.45822 0.508574   
## fval = 4667   
##   
## Outer iteration: 5   
## Min(hin): 2.409679e-05 Max(abs(heq)): 0.03948109   
## par: 5.33334 11.7062 6.31851 0.960519 0.987503 1.33335 2.83335 2.31896 1.16667 0.333337 3.36068 0.347735   
## fval = 4665   
##   
## Outer iteration: 6   
## Min(hin): 1.398227e-07 Max(abs(heq)): 0.002417662   
## par: 5.33333 11.6691 6.32816 0.997582 1.00134 1.33334 2.83334 2.33444 1.16667 0.333333 3.33717 0.332231   
## fval = 4665   
##   
## Outer iteration: 7   
## Min(hin): 1.103315e-07 Max(abs(heq)): 0.002417662   
## par: 5.33333 11.6691 6.32816 0.997582 1.00134 1.33334 2.83334 2.33444 1.16667 0.333333 3.33717 0.332231   
## fval = 4665   
##   
## Outer iteration: 8   
## Min(hin): 8.084029e-08 Max(abs(heq)): 0.002417662   
## par: 5.33333 11.6691 6.32816 0.997582 1.00134 1.33334 2.83334 2.33444 1.16667 0.333333 3.33717 0.332231   
## fval = 4665   
##   
## Outer iteration: 9   
## Min(hin): 5.134909e-08 Max(abs(heq)): 0.002417662   
## par: 5.33333 11.6691 6.32816 0.997582 1.00134 1.33334 2.83334 2.33444 1.16667 0.333333 3.33717 0.332231   
## fval = 4665   
##   
## Outer iteration: 10   
## Min(hin): 2.185789e-08 Max(abs(heq)): 0.002417662   
## par: 5.33333 11.6691 6.32816 0.997582 1.00134 1.33334 2.83334 2.33444 1.16667 0.333333 3.33717 0.332231   
## fval = 4665   
##   
## Outer iteration: 11   
## Min(hin): 6.993448e-07 Max(abs(heq)): 0.002417662   
## par: 5.33333 11.6686 6.32943 0.998109 1.00105 1.33333 2.83333 2.33425 1.16667 0.333335 3.33619 0.332421   
## fval = 4665   
##   
## Outer iteration: 12   
## Min(hin): 1.873813e-06 Max(abs(heq)): 0.002417662   
## par: 5.33334 11.6683 6.33007 0.998387 1.00089 1.33333 2.83333 2.33409 1.16667 0.333334 3.33573 0.332575   
## fval = 4665   
##   
## Outer iteration: 13   
## Min(hin): 6.711228e-07 Max(abs(heq)): 0.0005989608   
## par: 5.33338 11.6661 6.33607 1.0006 0.999557 1.33333 2.83333 2.33161 1.16667 0.333334 3.33104 0.335055   
## fval = 4665   
##   
## Outer iteration: 14   
## Min(hin): 8.288408e-07 Max(abs(heq)): 0.0005989608   
## par: 5.33334 11.6663 6.33479 1.00043 0.999682 1.33333 2.83333 2.33261 1.16673 0.333333 3.3322 0.33406   
## fval = 4665   
##   
## Outer iteration: 15   
## Min(hin): 0.0001342512 Max(abs(heq)): 4.096123e-05   
## par: 5.33364 11.6668 6.33795 0.999959 1.00003 1.33336 2.83422 2.32903 1.16676 0.352094 3.33002 0.337915   
## fval = 4668   
##   
## Outer iteration: 16   
## Min(hin): 3.411498e-06 Max(abs(heq)): 4.096123e-05   
## par: 5.33334 11.6667 6.33365 0.999981 1.00001 1.33333 2.83336 2.33291 1.16667 0.333716 3.33304 0.333759   
## fval = 4665   
##   
## Outer iteration: 17   
## Min(hin): 1.198593e-07 Max(abs(heq)): 1.891101e-06   
## par: 5.33333 11.6667 6.33336 0.999998 1 1.33333 2.83333 2.33329 1.16667 0.333341 3.33331 0.333373   
## fval = 4665   
##   
## Outer iteration: 18   
## Min(hin): 1.319238e-08 Max(abs(heq)): 2.491962e-08   
## par: 5.33333 11.6667 6.33334 1 1 1.33333 2.83333 2.33333 1.16667 0.333334 3.33333 0.333337   
## fval = 4665   
##   
## Outer iteration: 19   
## Min(hin): 9.083436e-10 Max(abs(heq)): 2.491962e-08   
## par: 5.33333 11.6667 6.33333 1 1 1.33333 2.83333 2.33333 1.16667 0.333333 3.33333 0.333334   
## fval = 4665   
##   
## Outer iteration: 20   
## Min(hin): 6.71136e-10 Max(abs(heq)): 2.491962e-08   
## par: 5.33333 11.6667 6.33333 1 1 1.33333 2.83333 2.33333 1.16667 0.333333 3.33333 0.333334   
## fval = 4665   
##   
## Outer iteration: 21   
## Min(hin): 4.129106e-10 Max(abs(heq)): 2.491962e-08   
## par: 5.33333 11.6667 6.33333 1 1 1.33333 2.83333 2.33333 1.16667 0.333333 3.33333 0.333334   
## fval = 4665   
##   
## Outer iteration: 22   
## Min(hin): 2.003731e-11 Max(abs(heq)): 2.491962e-08   
## par: 5.33333 11.6667 6.33333 1 1 1.33333 2.83333 2.33333 1.16667 0.333333 3.33333 0.333334   
## fval = 4665   
##   
## Outer iteration: 23   
## Min(hin): 3.02764e-10 Max(abs(heq)): 6.778067e-11   
## par: 5.33333 11.6667 6.33333 1 1 1.33333 2.83333 2.33333 1.16667 0.333333 3.33333 0.333334   
## fval = 4665   
##   
## Outer iteration: 24   
## Min(hin): 2.933271e-10 Max(abs(heq)): 7.340906e-12   
## par: 5.33333 11.6667 6.33333 1 1 1.33333 2.83333 2.33333 1.16667 0.333333 3.33333 0.333334   
## fval = 4665   
##

k$par <- ceiling(k$par)  
print(k$par)

## [1] 6 12 7 1 1 2 3 3 2 1 4 1

print(f(k$par))

## [1] 5490

# E & F  
f <- function(x) {  
 return(60\*sum(x[1:5]) + 60\*(x[1]+x[2]+x[3]+x[6]) + 90\*(x[4]+x[5]+x[6]+2\*x[7]))  
}

**Due to a preference among full-time agents to avoid late evening shifts, Alex can find only one qualified English-speaking agent willing to start work at 1 P.M. and 3 P.M.:**

**Part C:  
How many full-time English-speaking agents, full-time Spanish-speaking agents, and part-time agents should Alex hire under this new constraint?**

Full-time English-speaking agents: 6, 13, 7, 1, 1, 0, 0

Full-time Spanish-speaking agents: 2, 3, 3, 2, 1, 0, 0

Part-time English-speaking agents: 0, 0, 0, 0, 0, 4, 1

**Part D:  
What is the minimum cost under this new constraint?**

Minimum Cost: $1005.00 (approximately)

**PART E AND F**

# E AND F -----------------------------------------------------------------  
  
g <- function(x) {  
 h <- numeric(14)  
 h[1] <- 6\*x[1]-40  
 h[2] <- 6\*x[2]-85  
 h[3] <- 6\*(x[1]+x[3])-70  
 h[4] <- 6\*(x[2]+x[4])-95  
 h[5] <- 6\*(x[3] + x[5]+x[6])-80  
 h[6] <- 6\*(x[4] + x[6]+x[7])-35  
 h[7] <- 6\*(x[5] + x[7])-10  
 h[8:14] <- x[1:7]  
 return(h)  
}  
  
p0 <- rep(20, 7)  
k <- constrOptim.nl(p0, f, hin=g)

## Min(hin): 20   
## par: 20 20 20 20 20 20 20   
## fval: 19800   
## Min(hin): 0.05428387   
## par: 6.831598 14.42642 6.194848 1.710987 2.463819 5.252283 0.05428387   
## fval: 4718.179   
## Min(hin): 8.925508e-05   
## par: 6.66829 14.55065 7.045217 1.285673 1.699955 4.593857 8.925508e-05   
## fval: 4528.638   
## Min(hin): 2.245092e-07   
## par: 6.666682 14.56226 7.101349 1.271102 1.66796 4.564073 2.245092e-07   
## fval: 4525.106   
## Min(hin): 4.109929e-08   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 4.109929e-08   
## fval: 4525.017   
## Min(hin): 1.160809e-08   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 1.160809e-08   
## fval: 4525.017   
## Min(hin): 5.70985e-09   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 5.70985e-09   
## fval: 4525.017   
## Min(hin): 4.530202e-09   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 4.530202e-09   
## fval: 4525.017   
## Min(hin): 3.350554e-09   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 3.350554e-09   
## fval: 4525.017   
## Min(hin): 2.170906e-09   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 2.170906e-09   
## fval: 4525.017   
## Min(hin): 9.912578e-10   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 9.912578e-10   
## fval: 4525.017   
## Min(hin): 7.553282e-10   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 7.553282e-10   
## fval: 4525.017   
## Min(hin): 5.193986e-10   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 5.193986e-10   
## fval: 4525.017   
## Min(hin): 2.83469e-10   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 2.83469e-10   
## fval: 4525.017   
## Min(hin): 4.753937e-11   
## par: 6.666667 14.56281 7.103319 1.270525 1.666867 4.563151 4.753937e-11   
## fval: 4525.017

k$par <- ceiling(k$par)  
print(k$par)

## [1] 7 15 8 2 2 5 1

print(f(k$par))

## [1] 5130

**If all agents are bilingual:**

**Part E:  
How many full-time and part-time agents should Alex hire for each 2-hour shift to minimize operating costs while attending to all calls?**

Full-time agents (bilingual): 7, 15, 8, 2, 2, 0, 0

Part-time agents (bilingual): 0, 0, 0, 0, 5, 1, 0

**Part F (formerly "c" in the bilingual scenario):  
What is the minimum cost under the bilingual scenario?**

Minimum Cost: $4525.00 (approximately)

##Wage Rate:   
rate <- ((5610 - 5130) / 5610)  
percentage <- round(rate \* 100, 1)  
print(sprintf("%s%%", percentage))

## [1] "8.6%"

**Part G (formerly "d" in the bilingual scenario):  
What is the maximum percentage increase in the hourly wage rate that Alex can offer to bilingual agents over monolingual agents without increasing the total operating costs?**

Maximum Wage Increase: 8.6%